

# A Robust Fault Detection and Isolation Filter for a Horizontal Axis Variable Speed Wind Turbine

B. Svetozarevic, P. Mohajerin Esfahani, M. Kamgarpour, and J. Lygeros

**Abstract**—We develop a robust fault detection and isolation (FDI) technique in the presence of measurement noise and apply it to the horizontal axis variable speed wind turbine. In the first part, we provide a nonlinear model of the wind turbine in the form of differential-algebraic equations. We consider wind as a disturbance to the system, having two components: the wind speed and the wind direction. In the second part, treating the nonlinear term due to wind in the system dynamics as an unknown disturbance, we propose an optimization-based approach to robustify a linear residual generator with respect to measurement noise. The contribution of the noise into the residual is introduced in the framework of linear matrix inequalities, in which the requirements of the FDI filter are modeled as linear constraints. We illustrate the performance of our proposed method on the wind turbine benchmark model implemented in the FAST simulation code.

## I. INTRODUCTION

Wind turbines contribute to a growing part of the world's power production. The high penetration of wind turbines in the power grid gives rise to new challenges of increasing their reliability while reducing their operational and maintenance costs. Recently, these challenges have become more crucial as wind turbines are installed in remote locations such as offshore. One step in addressing the challenges is to introduce fault tolerant control (FTC), that is, to prevent faults from developing into failures by taking appropriate actions. An essential part of an FTC system is the fault detection and isolation (FDI) system [1], the design of which is the focus of this paper.

In the past few years several approaches on model-based FDI for wind turbines have appeared. A survey of these methods is provided in [3]. In 2009 Odgaard et al. set up a wind turbine benchmark model [11] for simulation of fault detection and accommodation schemes. They also set a competition that drew a series of papers. A summary of the results of these papers are collected in [9]. Recently, Odgaard and Johnson proposed the second benchmark model [10] with the aim to make the results of FDI systems more applicable to the wind power industry. We will use the second wind turbine benchmark model as a test model for our proposed FDI systems.

The model of the wind turbine is available in the FAST code [6]. FAST is a comprehensive aeroelastic simulator

which accounts for aerodynamics, mechanical structure, fatigue, and turbulence of the wind turbine [6]. Although FAST model is of great detail, it does not have a closed analytic form and thus cannot be used with model-based FDI filter design techniques. To overcome this, we develop an accurate model of horizontal axis variable speed wind turbine in the form of nonlinear differential-algebraic equations (DAE) based on wind turbine dynamics and the actuators and sensors provided in the FAST model. The FDI filter designed for this nonlinear model will also be tested on the more detailed FAST model.

For the state-of-the-art FDI for the class of DAE models we refer to the work of Nyberg et al. [8] where the residual generators are characterized as a set of matrix polynomial equations. Recent work by Mohajerin Esfahani et al. [7] proposed an optimization based approach of Nyberg's characterization to address the sensitivity of residuals to faults and to expand the class of models to nonlinear systems. Although these approaches provide provable guarantees on fault detection, unfortunately, they assume noiseless measurements. Measurement noise is an inherent part of the wind turbine sensors, as well as many other realistic engineering applications. In this article, following [7], we develop an optimization framework to robustify the residuals of the filter to measurement noise. The problem can be cast as minimization of  $\mathcal{H}_2$  norm of the transfer function from the measurement noise to the residual, which can be written in the form of linear matrix inequalities (LMIs) and solved effectively. Our FDI filter is implemented on the FAST model and the fault detection and isolation is validated for the faults provided in the benchmark paper [10].

This article is organized as follows. In Section II we develop a nonlinear state-space model of the horizontal axis variable speed wind turbine, describe the controllers used in the FAST model and the fault signals to be detected. In Section III we briefly review the FDI design scheme developed in [7] (approach I) and develop an optimization-based approach to the FDI filter design which minimizes the contribution of measurement noise to the filter residuals (approach II). In Section IV we apply the two approaches to the developed DAE wind turbine model. In Section V, simulation results of applying the FDI filters designed based on the two approaches to the wind turbine model in FAST are shown. Finally, in Section VI we conclude and provide directions for future research.

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## II. MODEL DEVELOPMENT

We consider a generic, three-blade, horizontal axis variable speed, pitch and yaw regulated 4.8 MW wind turbine [5]. We develop a wind turbine model of reduced complexity compared to FAST in the form of nonlinear differential-algebraic equations. The model takes into account accurate dynamics of the wind turbine, actuators and sensors as well as the controller structure developed based on FAST.

### A. Wind Turbine Dynamics

We model wind as having two components: wind speed and wind direction. Both components are accessible in FAST through appropriate sensors. The model of the wind turbine describes the conversion from wind power to mechanical and eventually electrical power. Here, we describe modeling of different components of the wind turbine: aerodynamics, drive train, generator and converter, pitch and yaw actuators, and sensors. Our model is an extension of the model given in [2] in that we consider both pitch and yaw angle actuation and introduce appropriate models of actuators.

1) *Aerodynamics*: The aerodynamics of the wind turbine determines the torque acting on the blades. In the case of pitch and yaw regulated turbine, the aerodynamic torque depends on both the pitch and yaw angles. Let  $\varepsilon_1(t)$  represent the yaw angle,  $d_w(t)$  the measured wind direction, and  $C_\varepsilon$  the scaling factor. To model the effect of yaw angle, we extend the expression for the aerodynamic torque for the pitch actuated turbine, given in [4, p.74], by including the term  $\cos((\varepsilon_1 - d_w)/C_\varepsilon)$ , given in [12, eq.(1)], which models the loss in the extracted power due to the deviations in the yaw angle. The aerodynamic torque for the pitch and yaw actuated wind turbine can then be written as

$$\tau_r = \sum_{i=1}^3 \frac{\rho \pi R^3 C_p(\lambda, \beta_{i1})}{6J_r} \cos\left(\frac{\varepsilon_1 - d_w}{C_\varepsilon}\right) v_w^2. \quad (1)$$

In the above, index  $i$  denotes the  $i^{\text{th}}$  blade,  $\rho$  is the air density,  $R$  is the radius of the blades, and  $C_p(\lambda, \beta_{i1})$  is the so called efficiency coefficient. The latter is a function of the tip speed ratio  $\lambda = \frac{\omega_r R}{v_w}$  and the pitch angle  $\beta_{i1}$ . Typically,  $C_p$  is given by numerical look-up tables. In this paper a standard nonlinear analytical expression is used [14]:

$$C_p(\lambda, \beta_{i1}) = 0.22 \left( \frac{116}{\lambda_t} - 0.4\beta_{i1} - 5 \right) e^{-\frac{12.5}{\lambda_t}},$$

$$\frac{1}{\lambda_t} \triangleq \frac{1}{\lambda + 0.08\beta_{i1}} - \frac{0.035}{\beta_{i1}^3 + 1}.$$

This term is the main source of nonlinearity in the model.

2) *Drive Train Model*: The conversion from the mechanical energy stored in the rotating blades (angular speed  $\omega_r$ , inertia  $J_r$ ) to electrical power is carried out via a flexible drive train with torsion stiffness constant  $K_d$ , and torsion damping constant  $B_d$ , and a gearbox with gear ratio  $N_g$ . The gearbox is used to step up the slow speed of the rotor to higher values at the generator side (angular speed  $\omega_g$ , inertial  $J_g$ ). Figure 1 shows the schematic diagram of the turbine mechanics. The torque at the rotor side is defined by Equation (1). By applying the Newton's law on that model,

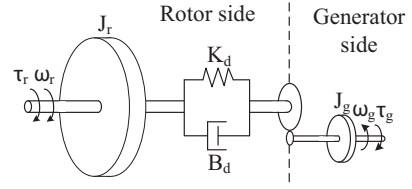


Fig. 1. Schematic diagram of the wind turbine mechanics

one can get the differential equations of  $\omega_r$  and  $\omega_g$ , as derived in Equations (2a) and (2b). The dynamics of the torsion  $\delta$  of the flexible drive train is defined in [2] and written in Equation (2c). The generator output power is given as  $P_g(t) = \eta_g \omega_g(t) \tau_g(t)$ , where  $\eta_g$  is the generator efficiency.

3) *Generator and Converter Actuator Model*: The generator and converter actuators are modeled in [10] as a closed loop first order transfer function  $\frac{\tau_g(s)}{\tau_{g,r}(s)} = \frac{\alpha_{gc}}{s + \alpha_{gc}}$ , where  $\alpha_{gc}$  is the generator and torque coefficient and  $\tau_{g,r}$  is the reference generator torque.

4) *Pitch Actuator Model*: The hydraulic pitch system is modeled in [10] as a closed loop second order transfer function

$$\frac{\beta(s)}{\beta_r(s)} = \frac{\omega_b^2}{s^2 + 2\zeta_b \omega_b s + \omega_b^2},$$

where  $\omega_b$  is the natural frequency and  $\zeta_b$  is the damping factor of the hydraulic actuator and  $\beta_r$  is the reference pitch angle.

5) *Yaw Actuator Model*: The yaw actuator is modeled in [6] as a closed loop second order transfer function

$$\frac{\varepsilon_1(s)}{\varepsilon_r(s)} = \frac{2\zeta_\varepsilon \omega_\varepsilon s + \omega_\varepsilon^2}{s^2 + 2\zeta_\varepsilon \omega_\varepsilon s + \omega_\varepsilon^2},$$

where  $\omega_\varepsilon$  and  $\zeta_\varepsilon$  are the natural frequency and the damping factor of the yaw actuator.

6) *Sensors*: The sensors in FAST provide measurements of state variables with an additive noise. The list of sensors available in the FAST model that we consider is given in Table I. The corresponding estimated noise powers are available in [10, Section III].

7) *Overall model*: The state space model of the horizontal axis variable speed, pitch and yaw regulated wind turbine is given by the following equations:

$$\dot{\omega}_r = \sum_{i=1}^3 \frac{\rho \pi R^3 C_p(\lambda, \beta_{i1})}{6J_r} \cos\left(\frac{\varepsilon_1 - d_w}{C_\varepsilon}\right) v_w^2 - \frac{B_d}{J_r} \omega_r + \frac{B_d}{N_g J_r} \omega_g - \frac{K_d}{J_r} \delta \quad (2a)$$

$$\dot{\omega}_g = \frac{\eta_d B_d}{N_g J_g} \omega_r - \frac{\eta_d B_d}{N_g^2 J_g} \omega_g + \frac{\eta_d K_d}{N_g J_g} \delta - \frac{1}{J_g} \tau_g \quad (2b)$$

$$\dot{\delta} = \omega_r - \frac{\omega_g}{N_g} \quad (2c)$$

$$\dot{\tau}_g = -\alpha_{gc} \tau_g + \alpha_{gc} \tau_{g,r}(t) \quad (2d)$$

$$\dot{\beta}_{i1} = \beta_{i2}, \quad i = 1, 2, 3 \quad (2e)$$

$$\dot{\beta}_{i2} = -\omega_{bi}^2 \beta_{i1} - 2\zeta_{bi} \omega_{bi} \beta_{i2} + \omega_{bi}^2 \beta_r, \quad i = 1, 2, 3 \quad (2f)$$

$$\dot{\varepsilon}_1 = \varepsilon_2 + 2\zeta_\varepsilon \omega_\varepsilon \varepsilon_r \quad (2g)$$

$$\dot{\varepsilon}_2 = -\omega_\varepsilon^2 \varepsilon_1 - 2\zeta_\varepsilon \omega_\varepsilon \varepsilon_2 + \omega_\varepsilon^2 (1 - 4\zeta_\varepsilon^2) \varepsilon_r \quad (2h)$$

For brevity we provide definitions of state variables, inputs, disturbances, and outputs of the model in Table I. The numerical values of the model parameters (based on [5]) are given in Table II.

TABLE I  
DEFINITIONS OF STATES/INPUTS/DISTURBANCES/OUTPUTS OF THE  
WIND TURBINE MODEL

States	Description	Units
$\omega_r$	Angular rotor speed	[rad/s]
$\omega_g$	Generator rotor speed	[rad/s]
$\delta$	Torsion angle	[rad]
$\tau_g$	Generator torque	[Nm]
$\beta_{i1}, \beta_{i2}$	Pitch angle and ang. vel. of $i^{\text{th}}$ blade	[deg],[deg/s]
$\varepsilon_1, \varepsilon_2$	Yaw angle and angular velocity	[deg],[deg/s]
Inputs	Description	Units
$\beta_r$	Pitch angle reference	[deg]
$\tau_{g,r}$	Generator torque reference	[Nm]
$\varepsilon_r$	Yaw angle reference	[deg]
Disturbances	Description	Units
$v_w$	Wind speed	[m/s]
$d_w$	Wind direction	[deg]
Outputs	Description	Units
$v_{w,m}$	Wind speed measurement	[m/s]
$\omega_{r,m}$	Angular rotor speed meas.	[rad/s]
$\omega_{g,m}$	Generator rotor speed meas.	[rad/s]
$\tau_{g,m}$	Generator torque measurement	[Nm]
$P_{g,m}$	Generator power measurement	[MW]
$\beta_{i1,m}$	Meas. of the pitch angle of $i^{\text{th}}$ blade	[deg]
$\Xi_{e,m}$	Yaw error measurement	[deg]

### B. Controller

The wind turbine benchmark model of [10] provides controllers for pitch, torque, and yaw. The pitch angle commands are computed using proportional-derivative (PD) control on the error between the filtered generator speed and the rated generator speed [5, Sec. 7.3]. The torque controller is a nonlinear region-based feedback whose function depends on the operating point. The inputs to the torque controller are the generator speed error and commanded pitch angle values. A lookup table which describes this feedback controller is provided in [5, Sec. 7.2]. The yaw controller is an on/off controller which operates with a constant angular speed and the direction given by the sign of the yaw error [10, Sec. II-B].

### C. Modeling Faults

We consider the faults given by Table III, which are a subset of all faults in the second benchmark model [10]. On the right-hand column of the table, we indicate the time at which each fault occurs as provided in the benchmark competition [10]. Our FDI design approach is developed for additive faults. If a multiplicative fault,  $s$  occurs in a given variable  $x_i$  to result in the signal  $sx_i$ , then we consider their product  $sx_i$  as a fault signal  $f_i$ . This way, we are able to detect and isolate both additive and multiplicative faults of Table III.

TABLE II  
MODEL PARAMETERS

$R$	63	[m]	$\zeta_\varepsilon$	0.02	[-]
$J_r$	$11.8 \cdot 10^6$	[kg·m <sup>2</sup> ]	$C_\varepsilon$	1	[-]
$J_g$	534	[kg·m <sup>2</sup> ]	$\omega_\varepsilon$	3	[Hz]
$N_g$	97	[-]	$\omega_{r,nom}$	1.26	[rad/s]
$\eta_d$	0.97	[-]	$\omega_{g,nom}$	122.22	[rad/s]
$K_d$	$867.64 \cdot 10^6$	[kg·m <sup>2</sup> ]	$\omega_{r,min}$	0.72	[rad/s]
$B_d$	$6.22 \cdot 10^6$	[ $\frac{\text{kg}\cdot\text{m}^2}{\text{rad}\cdot\text{s}}$ ]	$\omega_{g,min}$	68.40	[rad/s]
$\alpha_{gc}$	50	[Hz]	$\beta_{i1,min}$	-2	[deg]
$\eta_g$	0.944	[-]	$\beta_{i1,max}$	90	[deg]
$\omega_{bi}$	11.11	[Hz]	$ \beta_{i2} _{max}$	8	[deg/s]
$\zeta_{bi}$	0.6	[-]			

## III. FAULT DETECTION SCHEMES

In this section, we first review the optimization-based approach for FDI filter design proposed in [7]. Then, motivated by the presence of noise in the wind turbine sensors, we modify the approach to minimize contribution of measurement noise in the fault detection and isolation signal.

### A. FDI Filter Design - Approach I

The class of general linear models for which the FDI filter is designed has the form [8],

$$H(p)x + L(p)z + F(p)f = 0, \quad (3)$$

where  $p$  is the derivative operator and  $H, L, F$  are polynomial matrices in the operator  $p$ . We assume that the vector signal  $x := x(\cdot) : \mathbb{R}_+ \rightarrow \mathbb{R}^{n_x}$  is a piece-wise continuous function. We denote the space of piece-wise continuous functions  $x$  by  $\mathcal{W}^{n_x}$ . Similarly, we assume that  $z$  and  $f$  are piece-wise continuous functions from  $\mathbb{R}_+$  into  $\mathbb{R}^{n_z}$  and  $\mathbb{R}^{n_f}$ , respectively, with the corresponding spaces of piece-wise continuous functions  $\mathcal{W}^{n_z}$  and  $\mathcal{W}^{n_f}$ .

In this model the vectors  $x, z,$  and  $f$  represent, respectively, all the unknown, known, and fault signals. Throughout the paper, we use  $p$  as a variable of a matrix when the matrix is viewed as an operator, e.g.,  $H(p)$ , and we shall use the complex variable  $s$  instead of  $p$  if it is meant to be polynomial matrix, e.g.,  $H(s)$ .

One can verify that a classical linear state space description

$$\begin{cases} G\dot{X}(t) = AX(t) + B_u u(t) + B_d d(t) + B_f f(t) \\ Y(t) = CX(t) + D_u u(t) + D_d d(t) + D_f f(t) \end{cases} \quad (4)$$

TABLE III  
DEFINITION OF FAULTS

No.	Fault	Type	Time
Sensor faults			
1	Generator speed sensor	Scaling	130-150s
2	Pitch angle sensor	Stuck	185-210s
Actuator faults			
3	Pitch actuator	Slow change in dynamics	350-410s
4	Pitch actuator	Abrupt change in dyn.	440-465s

is a particular case of the linear model (3) by defining the vector signals  $x := \begin{bmatrix} X \\ d \end{bmatrix}$  and  $z := \begin{bmatrix} Y \\ u \end{bmatrix}$ , and matrices

$$H(p) := \begin{bmatrix} -pG + A & B_d \\ C & D_d \end{bmatrix}, \quad L(p) := \begin{bmatrix} 0 & B_u \\ -I & D_u \end{bmatrix},$$

$$F(p) := \begin{bmatrix} B_f \\ D_f \end{bmatrix}.$$

Let us denote the set of all behaviors of (3) in the absence of the fault signal  $f$  as follows:

$$\mathcal{M} := \{z \in \mathcal{W}^{n_z} \mid \exists x \in \mathcal{W}^{n_x} : H(p)x + L(p)z = 0\}.$$

**Definition 1.** (Sensitive Residual Generator): *In model (3), a proper linear time invariant filter  $r := R(p)z$  is a residual generator sensitive to fault  $f$  if the transfer function from  $f$  to  $r$  is nonzero, and for all  $z \in \mathcal{M}$  it holds that  $\lim_{t \rightarrow \infty} r(t) = 0$ .*

As shown in [8], a residual generator in the sense of Definition 1 can be expressed as

$$R(p) = a^{-1}(p)N(p)L(p), \quad (5)$$

where the polynomial matrix  $N(p)$  satisfies the following set of equations:

$$N(p)H(p) = 0, \quad (6a)$$

$$N(p)F(p) \neq 0, \quad (6b)$$

and  $a(p)$  is a polynomial of sufficiently high order whose roots have negative real part. Since the denominator polynomial of the filter (5),  $a(p)$ , is chosen *a priori*, our approach can be seen as selecting zeros of the transfer functions of the filter (5). The nontrivial matrix polynomial equations (6) can be written in a linear programming (LP) framework as proposed by the following lemma:

**Lemma 2.** [7, Section III-A] *Let  $N(s)$  be the solution of (6), where*

$$H(s) := \sum_{i=0}^{d_H} H_i s^i, \quad F(s) := \sum_{i=0}^{d_F} F_i s^i, \quad N(s) := \sum_{i=0}^{d_N} N_i s^i.$$

*In the above,  $d_H$ ,  $d_F$ , and  $d_N$  are the highest degrees of polynomials in  $H(p)$ ,  $F(p)$ , and  $N(p)$ , respectively. Then the conditions in (6) can equivalently be written as*

$$\bar{N}\bar{H} = 0, \quad (7a)$$

$$\|\bar{N}\bar{F}\|_{\infty} \geq 1, \quad (7b)$$

where  $\|\cdot\|_{\infty}$  denotes the infinity norm, and

$$\bar{N} := [N_0 \quad N_1 \quad \cdots \quad N_{d_N}],$$

$$\bar{H} := \begin{bmatrix} H_0 & H_1 & \cdots & H_{d_H} & 0 & \cdots & 0 \\ 0 & H_0 & H_1 & \cdots & H_{d_H} & 0 & \vdots \\ \vdots & & \ddots & \ddots & & \ddots & 0 \\ 0 & \cdots & 0 & H_0 & H_1 & \cdots & H_{d_H} \end{bmatrix},$$

$$\bar{F} := \begin{bmatrix} F_0 & F_1 & \cdots & F_{d_F} & 0 & \cdots & 0 \\ 0 & F_0 & F_1 & \cdots & F_{d_F} & 0 & \vdots \\ \vdots & & \ddots & \ddots & & \ddots & 0 \\ 0 & \cdots & 0 & F_0 & F_1 & \cdots & F_{d_F} \end{bmatrix}.$$

*Remark 3.* The inequality (7b) is not an LP. However, as explained in [7, Remark 3.2], the formulation of (7) can effectively be treated as several LP problems where the number of LPs is proportional to the degree of the filter  $d_N$ , which is a design parameter and is chosen *a priori*.

As shown in [7, Approach I], the contribution of the fault signal into the residuals can be maximized by solving the following optimization problem:

$$\begin{aligned} & \max_{\bar{N}} \|\bar{N}\bar{F}\|_{\infty} \\ & \text{s.t.} \begin{cases} \bar{N}\bar{H} = 0 \\ \|\bar{N}\|_{\infty} \leq 1 \end{cases} \end{aligned} \quad (8)$$

### B. FDI Filter Design - Approach II

In the residual generator design approach introduced in the previous subsection, the measurement noise was not taken into account. When there is noise in the measurement, its influence on the residual can be significant, as we shall show in Section V. To account for the measurement noise in the FDI filter design, we augment conditions (7) with additional conditions such that the contribution of measurement noise in the residual is minimized.

The impact of measurement noise on the FDI filter residual can be explained with the help of Figure 2. The FDI filter is fed by two known signals: the controller output,  $u(t)$ , and the measurements with noise,  $y(t)$ . Signal  $y_0(t)$  represents measurements without noise and  $r(t)$  is the residual. Note that measurement noise affects the FDI filter in two ways: as a direct input to the filter and as a signal filtered through the controller. Block  $W$  is a constant gain from the white noise signal  $e(t)$  to the measurement noise  $n(t)$  modeling the level of noise at each sensor, block  $T$  selects some of the measurements to feed the controller, and block  $K$  represents the controller. In our proposed approach, one needs to approximate the controller transfer function  $K$  via a static matrix gain. Hence, hereafter we assume that block  $K$  is a constant matrix, which in our application can be approximated based on simulation results in FAST.

It is a well-known result that the minimization of the  $\mathcal{H}_2$  norm of a transfer function of a given system corresponds to minimization of the variance of its output when the system is driven by a white noise [13]. Therefore, minimizing the contribution of measurement noise in the residual can be cast as minimization of the  $\mathcal{H}_2$  norm of the transfer function from the white noise signal to the residual.

Let us introduce a compact notation as in [13] to denote the transfer function  $G(s) = D + C(sI - A)^{-1}B$  by  $[\underline{A} \mid \underline{B}]$ . Suppose the state-space representation of the filter transfer function (5) is  $R(s) := [\frac{A_R}{C_R} \mid \frac{B_R}{D_R}]$ . Thus, the transfer function from the white noise signal  $e(t)$  to the residual  $r(t)$  has the following form:  $G(s) = [\frac{A_R}{C_R} \mid \frac{B_R M}{D_R M}]$ , where  $M$  is the constant gain matrix from  $e(t)$  to the input signals of the filter,  $u(t)$  and  $y(t)$ , defined as  $M = [\frac{K T W}{W}]$ .

Recall from subsection III-A that the target of the design is the numerator of the rational transfer functions of filter (5). Therefore, if we derive a state space representation of filter (5) in the observable canonical form, then the numerator

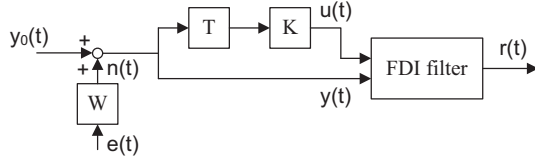


Fig. 2. Contribution of measurement noise in the residual

of the transfer function can be translated through the  $B_R$  component. Hence, the decision variables are expressed by  $B_R$ , and are related to the numerator of the FDI filter (5), given by  $\bar{N}$  in Lemma 2, as

$$B_{R,i} = \bar{N}L_i, \quad i = 1, \dots, d_N.$$

where  $B_{R,i}$  denotes the  $i$ -th row of matrix  $B_R$  and  $L_i$  denotes a matrix with  $i$ -th row block equal  $L$  and other rows equal zero.

The condition to bound the  $\mathcal{H}_2$  norm of the transfer function  $G(s)$  can be written as the following set of linear matrix inequalities (LMIs) [13, Section III.C]:

$$\begin{aligned} \|G(s)\|_2^2 \leq \lambda &\Leftrightarrow \\ \exists Q \succ 0, &\begin{bmatrix} QA_R^T + A_RQ & B_RM \\ M^T B_R & I \end{bmatrix} < 0, \\ \text{Trace}(C_RQC_R^T) &< \lambda. \end{aligned}$$

Now, to consider the contribution of measurement noise for an FDI filter in the sense of Definitions 1, we formulate an optimization problem with the objective concerned with the minimization of the  $\mathcal{H}_2$  norm of the transfer function  $G(s)$ :

$$\begin{aligned} \min_{\lambda, \bar{N}, Q, B_R} &\lambda & (9) \\ \text{s.t.} &\begin{cases} \bar{N}\bar{H} = 0 \\ \|\bar{N}\bar{F}\|_\infty \geq 1 \\ B_{R,i} = \bar{N}L_i, \quad i = 1, \dots, d_N \\ \begin{bmatrix} QA_R^T + A_RQ & B_RM \\ M^T B_R & I \end{bmatrix} < 0 \\ \text{Trace}(C_RQC_R^T) < \lambda \\ Q \in \mathcal{S}_{++}^n \end{cases} \end{cases}$$

where  $\mathcal{S}_{++}^n$  is the cone of symmetric positive definite matrices. Note that the constraints in (9) are not entirely linear in the decision variables, but nevertheless, the optimization problem (9) can be effectively solved as several semidefinite programming (SDP) problems. For the required number of SDPs to solve, we refer to Remark 3 concerning the same issue in approach I.

#### IV. FDI FILTERS FOR THE WIND TURBINE

In this section, the two FDI filter design techniques, described in subsections III-A and III-B, are applied to the wind turbine model developed in Section II.

The wind turbine model (2) and the fault signals given in Table III, can be compactly written in the state space notation (4), as follows:  $X(t) =$

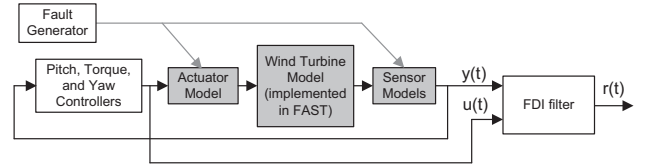


Fig. 3. Block diagram of major elements of the Simulink-based FDI model

$[\omega_r \ \omega_g \ \delta \ \tau_g \ \beta_{11} \ \beta_{12} \ \beta_{21} \ \beta_{22} \ \beta_{31} \ \beta_{32} \ \varepsilon_1 \ \varepsilon_2]^T \in \mathbb{R}^{n_x}$  are states,  $u(t) = [\beta_r \ \tau_{g,r} \ \varepsilon_r]^T \in \mathbb{R}^{n_u}$  are inputs,  $Y(t) = [v_{w,m} \ w_{r,m} \ w_{g,m} \ \tau_{g,m} \ P_{g,m} \ \beta_{1,m} \ \beta_{2,m} \ \beta_{3,m} \ \Xi_{e,m}]^T \in \mathbb{R}^{n_y}$  are outputs,  $d(t) = [v_w \ d_w \ d_{nl}]^T \in \mathbb{R}^{n_d}$  are unknown disturbances,  $f(t) = [f_1(t) \ f_2(t) \ f_3(t) \ f_4(t)]$  corresponds to the fault signals we aim to detect. Here, the disturbance term  $d_{nl}$  denotes the nonlinearity due to wind as described in equation (1). Thus, the only nonlinear term in the turbine dynamics (2a) which is a function of the wind is treated as an unknown disturbance. The matrices  $A \in \mathbb{R}^{n_x \times n_x}$ ,  $B_u \in \mathbb{R}^{n_x \times n_u}$ ,  $B_d \in \mathbb{R}^{n_x \times n_d}$ ,  $B_f \in \mathbb{R}^{n_x \times n_f}$ ,  $C \in \mathbb{R}^{n_y \times n_x}$ ,  $D_d \in \mathbb{R}^{n_y \times n_d}$ , and  $D_f \in \mathbb{R}^{n_y \times n_f}$  are obtained from the state-space model of subsection III-A, where  $n_x = 12$ ,  $n_u = 3$ ,  $n_y = 9$ ,  $n_d = 3$ , and  $n_f = 4$ .

The next step in approach II to FDI filter design is to estimate the variances of noises in each measurement. The measurement noise levels can be obtained from the Simulink<sup>®</sup>-based model [10]. From the variances of the measurement noises, one can obtain the static gain matrices  $W$  and  $K$  as described in subsection III-B. Finally, for both approaches, we select some stable dynamics for the denominator of the filters (5). Then, by solving the LP program (8) and the SDP program (9), one obtains the zeros of the transfer functions of the FDI filter for approaches I and II, respectively.

The designed FDI filter is integrated with the second wind turbine benchmark model as shown in Figure 3. Since the control signal  $u(t)$  can be obtained by passing the corresponding measurements  $y(t)$  through the controller, both approaches to FDI (see sections III-A and III-B) can be applied as shown in Figure 2.

#### V. IMPLEMENTATION

We present the simulation results of integrating the fault detection and isolation schemes designed in the previous section to the second wind turbine benchmark model in FAST. In order to show the practical importance of addressing the measurement noise in the FDI filter design, we compare the performance of the two filters for a given fault as shown in Figure 4. Notice that the filter based on approach I cannot identify the fault due to high measurement noise, while the filter based on approach II successfully suppresses the measurement noise and detects the fault.

In Figure 5 the simulation results of applying approach II to design a bank of filters to detect and isolate the faults in FAST are shown. The top panel indicates the fault signals appearing consecutively in the benchmark problem in FAST simulation code. The four lower panels show the filter

residuals corresponding to faults 1, 2, 3, and 4 respectively. As anticipated, we observed that approach II works best if we can accurately estimate variances of the measurement noises and the static gain of the controllers. These gains could be better estimated and approximated for faults 1, 2 and 4 than fault 3 and thus the residual signal of fault 3 contains more noise.

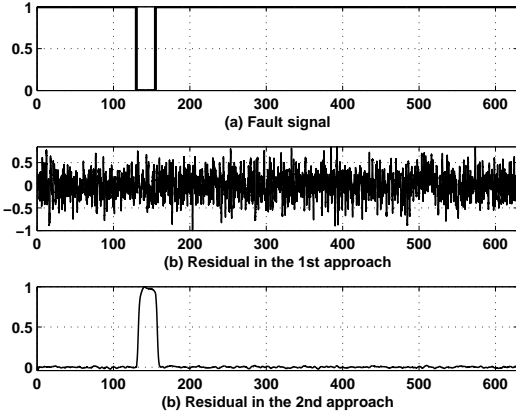


Fig. 4. Comparison of the approaches I and II to robust FDI filter design

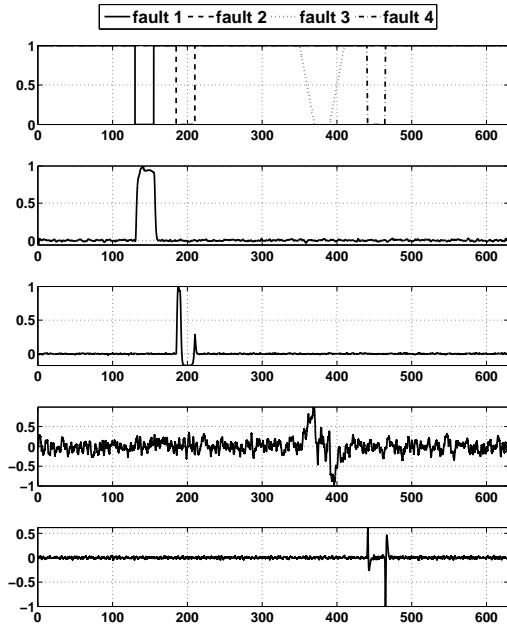


Fig. 5. Fault detection and isolation according to approach II - The top panel indicates the fault signals appearing consecutively. The four lower panels show the filter residuals corresponding to faults 1, 2, 3 and 4 respectively.

## VI. CONCLUSIONS

We developed a fault detection and isolation scheme for a general class of dynamical systems modeled by differential

algebraic equations. Our filter was designed by formulating an optimization problem which minimized the contribution of measurement noise while detecting the fault signal in the presence of unknown disturbances. We implemented our methodology for the wind turbine model provided in the FAST simulation environment. The wind turbine faults were successfully detected and isolated. In future, motivated by stochastic models of wind forecast, we plan to extend our approach to address stochastic disturbances and faults.

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