Model Based Power Optimisation of Wind Farms

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***Abstract*— We present a framework for maximising the power output of a wind farm considering wake effects. A parameter representing the total power coefficient of a wind farm is introduced, which quantifies the wind speed deficit due to wake. For any given wake model, this parameter is a function of the blade pitch angle and the tip speed ratio of the individual turbines, and is independent of the wind speed. Thus, the variables associated with each turbine can be optimised offline, given the wind farm layout, in order to determine reference set points that maximise power production. An MPC controller is designed for individual turbines to track the optimal reference set points. The performance is illustrated with the turbine simulator FAST and the wind farm simulator Aeolus SimWindFarm.**

I. INTRODUCTION

In 2012 the European Union (EU) reached 100 GW of wind power capacity, meeting the power needs of 57 million households as stated in the annual report of the European Wind Energy Association (EWEA) [1]. The EU aims to get 20% of its energy from renewable sources by 2020. The installation of wind farms and suitable control strategies are a key factor in achieving the targets.

Due to wake effects downstream wind turbines face a loss of wind power leading to a substantial power loss in wind farms. Models have been developed to calculate and simulate the wind speed experienced by each turbine in wind farms based on discretised wind field and Navier-Stokes equations [2], [3], or based on dynamic models of the wind turbines and the wake [4]-[6]. The validation and applicability of a wake model and the choice of its parameters depend on the site and terrain as investigated in [7]-[9].

Traditionally, feedback control has been used for tracking and optimisation of a single turbine power output. Recently, MPC approaches were applied to wind turbines for load mitigation [10]-[13], or to regard turbine constraints [14], [15]. These works often require current or future wind speed information which might be available from measurements such as LIDAR (light detection and ranging). Recent work [16] has shown that the use of MPC at high frequencies is possible enabling MPC application for online wind turbine control.

While single turbine control has been studied extensively, the control and optimisation of wind farms is a young field of research. In [17] a stationary model of the wind turbines is used to investigate the influence of upstream wind turbines on the power generation of downstream turbines. The publications [18]-[21] deal with tracking of a desired power output and load mitigation of wind farms. A model-free approach with a learning rule to maximise the power output of the farm is presented in [22]. The application of online optimisations in wind farms is difficult in reality as measurements of the total wind farm power are mainly influenced by the fast wind speed dynamics. Furthermore, the wake due to different control strategies is barely detectable as it takes several hundred seconds to spread in the whole farm.

In this work, we formulate and explore an approach for maximising the power output of a wind farm. The approach is based on deriving a single parameter that captures the total power coefficient of a wind farm, given an arbitrary wake model. The introduced total power coefficient is independent of wind speed and depends only on the wind speed deficit, which itself is a function of the blade pitch angle and the rotor speed of individual turbines. Thus, the variables associated with individual turbines can be determined using a static optimisation in order to minimise the deficit. The consideration of arbitrary wake model allows for appropriate wake model choice for a given site. An MPC controller is presented based on a 5 MW offshore wind turbine model [23] in order to track the set points of the optimisation. The formulation is verified using the well known wind farm simulators FAST (Fatigue, Aerodynamics, Structures, and Turbulence) [24] and Aeolus SimWindFarm [21].

The paper is organised as follows. In Section II we describe the wind turbine model and the general wake model. In Section III we present the objective and the algorithm for the wind farm optimisation leading to the optimal reference values for every turbine. The MPC turbine controller to regulate the turbine to the desired reference values is defined in Section IV. In Section V we illustrate the performance of the optimisation and the controller using the simulators FAST and Aeolus SimWindFarm.

II. WIND FARM MODEL

We consider a wind farm with n wind turbines denoted by the set $N = \{1,2,...,n\}$. For simplicity, we assume the wind speed $V_\infty(t)$ is uniform over the wind farm span and approaches the wind farm with a constant direction. Moreover, we assume that all wind turbines are oriented facing the wind with the rotor perpendicular to the wind direction. We state those simplifications without loss of generality as the yaw angles of the wind turbines are controlled independently of the other states. The optimisation for any wind direction is achieved by turning the coordinate system. The MPC controller for the wind turbine control is based on the dynamic turbine model (see Subsection II-A). For the wind farm optimisation we simplify the characterisation of every turbine $i \in N$ to its position $(s_i, r_i) \in \mathbb{R}^2$ and the reference values for the tip speed ratio $\lambda_{ri} \in \mathbb{R}_+$ and the pitch angle $\beta_{ri} \in [0, \pi]$ as elaborated in Subsection II-D.

A. Wind Turbine Model

Advanced simulators like FAST [24] are used to describe the dynamics of wind turbines. However, these models are too complex for implementation in an MPC scheme. Hence we develop a state space representation of the 5 MW offshore

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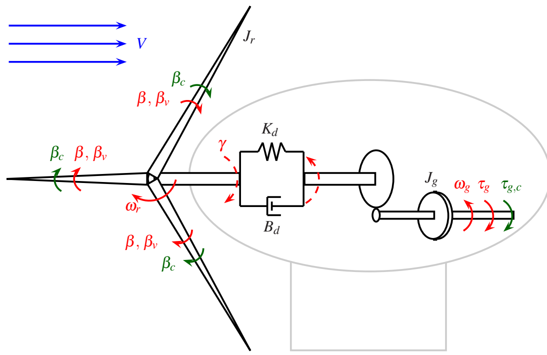


Fig. 1: States (red), inputs (green) and disturbance (blue) of the wind turbine.

R	63	[m]	$\omega_{r,nom}$	1.267	[rad/s]
J_r	$53.4 \cdot 10^6$	[$\mathrm{kg \cdot m^2}$]	$\omega_{r,min}$	0.723	[rad/s]
J_g	534	[$\mathrm{kg \cdot m^2}$]	β_{min}	0	[deg]
N_g	97	[-]	β_{max}	90	[deg]
η_d	0.97	[-]	β_{rate}	8	[deg/s]
K_d	$867.64 \cdot 10^6$	[$\mathrm{kg \cdot m^2}$]	$\tau_{g,min}$	0	[Nm]
B_d	$6.215 \cdot 10^6$	$\left[\frac{\mathrm{kg \cdot m^2}}{\mathrm{rad \cdot s}}\right]$	$\tau_{g,max}$	47403	[Nm]
α_{gc}	50	[Hz]	$\tau_{g,rate}$	15000	[Nm/s]
η_g	0.944	[-]	P_{nom}	5	[MW]
ω_b	11.11	[Hz]			
ζ_b	0.6	[-]			

TABLE I: Parameters of wind turbine

three-bladed upwind turbine [23] based on [25] and [21] with collective pitch control, meaning that every blade has the same reference value and dynamics. We neglect the states for the yaw angle due to the assumption of constant wind direction. The states, represented by $x \in \mathbb{R}^6$, are the angular rotor speed ω_r, angular generator speed ω_g, drive train torsion angle γ, generator torque τ_g, pitch angle β and pitch angular velocity β_v. The system is influenced by the wind speed faced at the turbine V as disturbance $d \in \mathbb{R}$ and the two inputs $u \in \mathbb{R}^2$ set by the controller, namely the generator torque input $\tau_{g,c}$ and the pitch angle input β_c. The measurable outputs of the system are the wind speed V_m and the generator speed $\omega_{g,m}$. The flexible drive train with torsion stiffness constant K_d, torsion damping constant B_d, drive train efficiency η_d and gear box ratio N_g transfers the power from the low speed shaft of the rotor with inertia J_r to the high speed shaft of the generator with the inertia J_g. Fig. 1 illustrates the states, inputs, disturbance and mechanics of the wind turbine. The whole system is driven by the aerodynamic power defined as $P_{aero} = \frac{1}{2}\rho\pi R^2 C_P(\lambda,\beta)V^3$ where V is the wind speed, $C_P(\lambda,\beta)$ the power coefficient, R the rotor radius and ρ the air density. We derive the values of the power coefficient C_P as an explicit function of the tip speed ratio $\lambda = \frac{\omega_r R}{V}$ and the pitch angle β. Dividing the aerodynamic power by the angular generator speed yields the moment introduced by the wind on the rotor as visible in (1). We model the dynamics of the generator as a first order closed loop transfer function $\frac{\tau_g(s)}{\tau_{g,c}(s)} = \frac{\alpha_{gc}}{s+\alpha_{gc}}$ using the generator and converter coefficient α_{gc} based on [25]. The dynamics of the hydraulic pitch system of the blades are represented by the closed loop second order transfer function $\frac{\beta(s)}{\beta_c(s)} = \frac{\omega_b^2}{s^2+2\zeta_b\omega_b+\omega_b^2}$ and the damping factor ζ_b. The following equations describe the state space model of the variable speed, collective pitch controlled wind turbine.

$$\dot{\omega}_r = \frac{1}{J_r}\left[\frac{\rho\pi R^2 C_P(\lambda,\beta)}{2\omega_r}V^3 - K_d\gamma - B_d\left(\omega_r - \frac{\omega_g}{N_g}\right)\right] \quad (1)$$

$$\dot{\omega}_g = \frac{1}{J_g}\left[-\tau_g + \frac{\eta_d}{N_g}\left(K_d\gamma + B_d\left(\omega_r - \frac{\omega_g}{N_g}\right)\right)\right] \quad (2)$$

$$\dot{\gamma} = \omega_r - \frac{\omega_g}{N_g} \quad (3)$$

$$\dot{\tau}_g = -\alpha_{gc}\tau_g + \alpha_{gc}\tau_{g,c} \quad (4)$$

$$\dot{\beta} = \beta_v, \quad (5)$$

$$\dot{\beta}_v = -\omega_b^2\beta - 2\zeta_b\omega_b\beta_v + \omega_b^2\beta_c \quad (6)$$

Considering the generator efficiency η_g the power output of the generator is

$$P_g = \eta_g\omega_g\tau_g. \quad (7)$$

Ideally, the turbine speed is limited by the minimal and nominal angular rotor speed $\omega_{r,min}$ and $\omega_{r,nom}$. Hard constraints exist for the pitch angle with β_{min} and β_{max}, the generator torque with $\tau_{g,min}$ and $\tau_{g,max}$ and the generator power with P_{nom} as stated in [23] and [25]. Furthermore, we need to consider the rate limits of the pitch angle and the generator torque β_{rate} and $\tau_{g,rate}$. Table I lists the parameters of the wind turbine.

B. Wake Model

Wake models are a simplified characterisation of the wake resulting from a single wind turbine. Throughout the paper we use a general formulation of wake effect to describe the wind velocity profile $V(\bar{s}_j,\bar{r}_j,C_{Tj})$ caused by a single turbine $j \in N$ of the form

$$V(\bar{s}_j,\bar{r}_j,C_{Tj}) = V_\infty\left[1 - \delta V(\bar{s}_j,\bar{r}_j,C_{Tj})\right],$$

where $\delta V(\bar{s}_j,\bar{r}_j,C_{Tj})$ is the fractional velocity deficit at the relative coordinate $(\bar{s},\bar{r})$ downstream of turbine j. Most wake models characterise the wake deficit caused by turbine j as a function of the thrust coefficient $C_{Tj}(\lambda_j,\beta_j)$ and downstream distance to the vertex in wind direction $\bar{s}_j = s - s_j$ and in orthogonal direction $\bar{r}_j = |r - r_j|$ as illustrated in Fig. 2. The thrust coefficient C_{Tj} is a function of the tip speed ratio λ_j and pitch angle β_j of turbine j taken from lookup tables. For the simulations presented in this work we adapt one of the oldest and simplest wake models developed by Jensen in 1983 [4] which is still used by the recent wind farm simulator Aeolus SimWindFarm [21]. It describes the wind velocity deficit as linear function of the thrust coefficient of the form

$$\delta V = \begin{cases} \frac{1}{2}C_{Tj}\left(1 + \frac{\bar{s}_j}{4R}\right)^{-1} & \text{if } \bar{r}_j \leq \sqrt{4R^2 + \bar{s}_j R} \\ 0 & \text{if } \bar{r}_j > \sqrt{4R^2 + \bar{s}_j R} \end{cases} \quad (8)$$

with the wake radius $R_w(\bar{s}_j,C_{Tj}) = \sqrt{4R^2 + \bar{s}_j R}$. Let us recall that the proposed approach in this work is not restricted to a specific model and the model (8) is just an example for our simulation results.

C. Wake Interaction Model

In wind farms, overlapping wakes from several wind turbines pose a core challenge to modelling. To derive the aggregate wind velocity deficit faced by each turbine $i \in N$, we define the set $W_i \in N$ of all upwind turbines causing a wake on turbine i. We describe the aggregate wind velocity

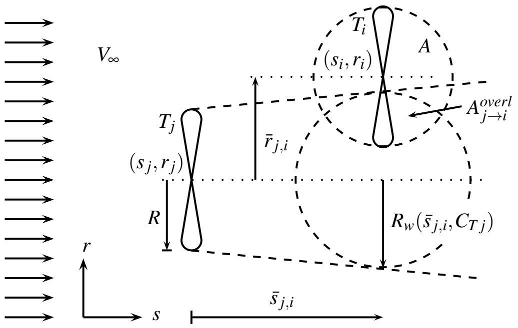


Fig. 2: Coordinates of wake and wake interaction model.

V_i at any turbine $i \in N$ as a function of the aggregate velocity deficit δV_i of the form

$$V_i = V_\infty \left[1 - \delta V_i \left((\bar{s}_{j,i}, \bar{r}_{j,i}, C_{Tj})_{j \in W_i} \right) \right]. \tag{9}$$

The aggregate velocity deficit of turbine i depends on the downstream distance to the vertex in wind direction $\bar{s}_{j,i} = s_i - s_j$ and orthogonal direction $\bar{r}_{j,i} = |r_i - r_j|$ and the thrust coefficient $C_{Tj}(\lambda_j, \beta_j)$ of every wind turbine $j \in W_i$.
As specific wake interaction model for the simulations we choose the commonly used approach of the momentum balance [5]. The aggregate wind velocity of turbine i is given by

$$\delta V_i = \sqrt{\sum_{j \in W_i} \left(\delta V_i (\bar{s}_{j,i}, \bar{r}_{j,i}, C_{Tj}) \frac{A^{overl}_{j \to i}}{A} \right)^2}$$

where $A = \pi R^2$ is the swept area of the rotor and $A^{overl}_{j \to i}$ is the area of the overlap between the wake generated by turbine j and the rotor swept area of turbine i (see Fig. 2).

D. Power Model

For the optimisation of a wind farm it is too complex to model all the states of the wind turbine and describe the power output with (7). Hence we characterise the power of turbine i by

$$P_i = \frac{1}{2} \rho \pi R^2 \eta C_{Pi} (\lambda_{ri}, \beta_{ri}) V_i^3 \tag{10}$$

as a function of the reference values for the tip speed ratio λ_{ri} and the pitch angle β_{ri} with $\eta = \eta_d \eta_g$ representing the overall turbine efficiency. We can influence the power P_i of turbine i with the two control parameters λ_{ri} and β_{ri}. Those are the reference points which the turbine needs to track. For a turbine operating in steady state at $\lambda_i = \lambda_{ri}$ and $\beta_i = \beta_{ri}$ (10) and (7) result in the same value. The total power generated by a wind farm is

$$P_{tot} = \sum_{i \in N} P_i (\lambda_{ri}, \beta_{ri}). \tag{11}$$

III. WIND FARM OPTIMISATION

The aim of the wind farm optimisation in our formulation is to maximise the power P_{tot} by setting the optimal reference values λ^*_{ri} and β^*_{ri} for every turbine $i \in N$. The MPC controller of every turbine tracks the desired reference values as illustrated in Fig. 3.

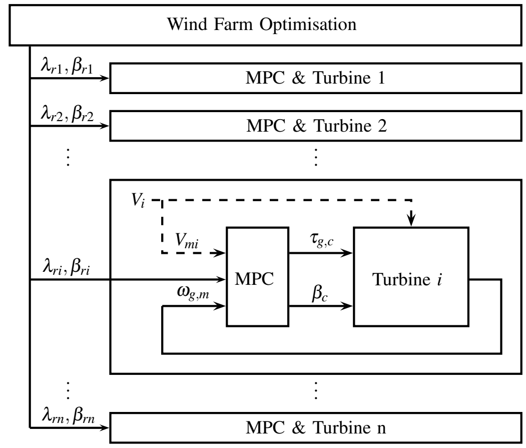


Fig. 3: Scheme of wind farm optimisation and single turbine control.

A. Objective

The total power of the wind farm depends on the wind speed. Since the wind speed cannot be influenced, we aim for an independent optimisation objective. Combining (10) and (11) yields the total power $P_{tot} = \sum_{i \in N} \frac{1}{2} \rho \pi R^2 \eta C_{Pi} V_i^3$ as a function of the wind V_i at every wind turbine i. We replace the wind speed V_i with the wake model in (9) and achieve the total power

$$P_{tot} = \eta \underbrace{\frac{1}{2} \rho \pi R^2 V_\infty^3}_{P_{wind}} \underbrace{\sum_{i \in N} C_{Pi} \left[1 - \delta V_i \left((\bar{s}_{j,i}, \bar{r}_{j,i}, C_{Tj})_{j \in W_i} \right) \right]^3}_{C_{P,tot}},$$

where δV_i is the aggregate velocity deficit of every turbine i. We simplify the total power P_{tot} to a product of the turbine efficiency η, the wind power of ambient wind speed V_∞ over the rotor area P_{wind} and the total power coefficient of the wind farm $C_{P,tot}$. The total power coefficient characterises the power production of a wind farm independent of the actual wind speed. The distances between the turbines $(\bar{s}_{j,i}, \bar{r}_{j,i})$ are determined by the wind farm layout. Hence, we introduce the optimisation objective with the reference tip speed ratio λ_{ri} and pitch angle β_{ri} for every turbine $i \in N$ as follows:

$$(\lambda^*_{ri}, \beta^*_{ri})_{i \in N} = \underset{(\lambda_{ri}, \beta_{ri})_{i \in N}}{\arg\max}\, C_{P,tot} \left((\lambda_{ri}, \beta_{ri})_{i \in N} \right),$$

where the total power coefficient $C_{P,tot}$ is defined as

$$C_{P,tot} \left((\lambda_{ri}, \beta_{ri})_{i \in N} \right) = \sum_{i \in N} C_{Pi} \left[1 - \delta V_i \left((\bar{s}_{j,i}, \bar{r}_{j,i}, C_{Tj})_{j \in W_i} \right) \right]^3$$

with the power and the thrust coefficient $C_{Pi}(\lambda_{ri}, \beta_{ri})$ and $C_{Ti}(\lambda_{ri}, \beta_{ri})$ as functions of the reference tip speed ratio and pitch angle.

B. Optimal Operating Points for a Given Layout

Consider n turbines in a wind farm with a given layout. The total power coefficient $C_{P,tot}$ depends on the reference values of all n turbines, a total of $2n$ optimisation parameters. For a large number of wind turbines the number of parameters lies beyond a computationally tractable brute force optimisation. To address this issue, we propose a heuristic algorithm to find the optimal set points. Consider an iterative algorithm that loops through every wind turbine $i \in N$ calculating for every step the total power coefficient of the whole wind farm $C_{P,tot}$ depending on just the reference values $(\lambda_{ri}, \beta_{ri})$ of the current turbine i while keeping the reference values $(\lambda_{rj}, \beta_{rj})_{j\in N\setminus i}$ of the other turbines constant. The algorithm is initialised with the reference values for greedy control $\lambda_{r,greedy} = 7.5$ and $\beta_{r,greedy} = 0°$ and terminates if two successive loops result in the same reference values for every turbine. We obtain the optimal total power coefficient depending on the reference values $(\lambda_{ri}, \beta_{ri})$ of a single turbine i using brute force with a reduced number of states. Notice that one may limit the range of possible optimal reference values around the greedy reference values $\lambda_{r,greedy}$ and $\beta_{r,greedy}$ thanks to the steep decay of the power coefficient $C_P(\lambda, \beta)$. The algorithm is presented below:

$(\lambda_{ri} \leftarrow 7.5)_{i\in N}$ {initialise λ_{ri}}
$(\beta_{ri} \leftarrow 0)_{i\in N}$ {initialise β_{ri}}
while $\varepsilon > 0$ **do**
  for $i = 1$ to n **do** {iterate over all turbines}
    $(\lambda_{ri}, \beta_{ri}) \leftarrow \underset{(\lambda_{ri},\beta_{ri})}{\operatorname{argmax}} C_{P,tot}\left((\lambda_{ri}, \beta_{ri})_{i\in N}\right)$
  end for
  $\varepsilon \leftarrow \sum_{i=1}^{n}\left(\lambda_{ri} - \lambda_{ri,old}\right)^2 + \left(\beta_{ri} - \beta_{ri,old}\right)^2$
  $\left(\lambda_{ri,old} \leftarrow \lambda_{ri}\right)_{i\in N}$
  $\left(\beta_{ri,old} \leftarrow \beta_{ri}\right)_{i\in N}$
end while

This procedure does not guarantee to find the global maximum of the total power coefficient $C_{P,tot}$, but the result is an improvement compared to the greedy reference values as will be seen in simulations. However, in practise the algorithm shows good performance and returned the global maximum, obtained with exhaustive search for up to 5 turbines in a row, within two to three iteration. The reason is that the optimal reference values of a turbine hardly depend on the reference values of the other turbines but mainly on the turbine's position which is constant.

IV. TURBINE MPC CONTROLLER

Traditional feedback controllers aim to regulate the wind turbine to the maximum of the power coefficient at the greedy reference values $\lambda_{r,greedy} = 7.5$ and $\beta_{r,greedy} = 0°$ [23]. For wind farm optimisation we set varying reference values λ_{ri}^* and β_{ri}^* for every turbine $i \in N$. The MPC controller sets the turbine inputs $\tau_{g,c}$ and β_c such that the captured power is optimised for below rated wind speeds and kept at the rated power for above rated wind speeds. This results in a region based MPC controller using information of the measured angular generator speed $\omega_{g,m}$ as the generally used feedback controllers and in addition the current wind speed measurement V_m which is measured by the anemometer for every time step and is assumed to be constant over the prediction horizon.

To guarantee convexity of the MPC optimisation we linearise the non linear state space equations (1)-(6). Since the number of states, inputs and disturbances of the wind turbine model is larger than the number of equations, we need measurements to determine the equilibrium point. The measured values are the angular speed of the generator $\omega_{g0} = \omega_{g,m}$ and the current wind speed $V_0 = V_m$. Furthermore, we set the inputs of the turbine the pitch angle β_0 and the torque of the generator τ_{g0} to the optimisation values β_c and $\tau_{g,c}$ of the last iteration. The other states of the equilibrium point, like $\gamma_0 = \frac{V_0^3 \rho \pi R^2 C_p(\lambda_0,\beta_0)}{2K_d \omega_r}$, can be obtained by setting $\dot{x} = 0$ leading to the linearised and discretised dynamics

$$\hat{x}(k+1) = A_k \hat{x}(k) + B_k \hat{u}(k), \tag{12}$$

where $\hat{x} = x - x_0$ and $\hat{u} = u - u_0$ describe the deviation from the equilibrium. In (12) the disturbance d is neglected as we assume constant wind speed over the prediction horizon. For every time step k the equilibrium point and the linearisation matrices A_k and B_k are updated according to the new measurements $\omega_{g,m}$ and V_m.

A. Reference Vector

The optimisation of the MPC minimises the deviation of the states from the reference vector. We determine the reference signal such that the wind turbine tracks the reference values λ_r^* and β_r^* provided by the wind farm optimisation framework as described in the preceding Section. The reference value for the rotor speed $\omega_{r,ref} = \frac{\lambda_r^* V_{LP}}{R}$ tracks the optimal operating point λ_r^* depending on the low pass filtered wind speed V_{LP}. The low pass filter with the time constant $T = 10\,\mathrm{s}$ is necessary because the dynamics of the wind V_m are much faster than the dynamics of the generator speed ω_g. The dynamics of the filter are $\dot{V}_{LP} = -\frac{1}{T}V_{LP} + \frac{1}{T}V_m$. Analogously, we set the reference value for the pitch angle to the optimal operating point $\beta_{ref} = \beta_r^*$. The reference for the generator speed is derived from the rotor speed reference $\omega_{g,ref} = N_g \omega_{r,ref}$. To maximise the power, the reference for the torque is defined as maximum torque of the generator regarding the nominal power P_{nom} of the turbine. The reference vector for the state vector $x = [\omega_r \quad \omega_g \quad \gamma \quad \tau_g \quad \beta \quad \beta_v]^\top$ includes the upper limit of the generator torque and the limits of the nominal and the cut-in rotor and generator speed $\omega_{g,nom} = N_g \omega_{r,nom}$ and $\omega_{g,min} = N_g \omega_{r,min}$. The reference vector is given by

$$x_{ref} = \begin{bmatrix} \max(\omega_{r,min}, \min(\omega_{r,ref}, \omega_{r,nom})) \\ \max(\omega_{g,min}, \min(\omega_{g,ref}, \omega_{g,nom})) \\ \gamma_0 \\ \min\left(\tau_{g,max}, \frac{P_{nom}}{\eta_g \omega_{g0}}\right) \\ \beta_{ref} \\ 0 \end{bmatrix}.$$

B. Optimisation

We formulate the optimisation as a linear MPC with quadratic cost with the prediction horizon of N_p steps at time step k_0 using the cost function

$$V(x_0) = \min_{\hat{u}} \sum_{k=k_0}^{k_0+N_p-1} \left[\tilde{x}^\top(k) Q_c \tilde{x}(k) + \hat{u}^\top(k) R_c \hat{u}(k)\right] + F_c$$

with respect to dynamic constraint

$$\tilde{x}(k+1) = A_k \tilde{x}(k) + B_k \hat{u}(k).$$

By minimising the deviation from the reference point $\tilde{x}(k) = x(k) - x_{ref}$ the turbine states are regulated to the reference values x_{ref}. The matrices Q_c and R_c define the weight of the states and inputs and F_c defines the terminal cost. Since we want to control the generator speed ω_g and pitch angle β to the optimal operating point $\omega_{g,ref}$ and β_{ref} we weigh those states. Some weight is also set on the torque τ_g to maximise the torque while still allowing tracking of the desired turbine speed $\omega_{g,ref}$. Testing several settings in simulations yields the weight matrices $Q_c = \mathrm{diag}\left(\begin{bmatrix}0 & 10 & 0 & 10^{-6} & 10 & 0\end{bmatrix}\right)$ and $R_c = \mathrm{diag}\left(\begin{bmatrix}10^{-5} & 1\end{bmatrix}\right)$. We formulate the terminal cost $F_c = x^\top(k_0+N_p)P_c x(k_0+N_p)$ by weighing the final state with the infinite horizon solution P_c of the associated discrete-time algebraic Riccati equation obtained for every time step as in [15].

The turbine is controlled with a region based MPC. The hard constraints for the system input $\hat{u} = \begin{bmatrix}\hat{\tau}_{g,c} & \hat{\beta}_c\end{bmatrix}^\top$ take into account the absolute and rate limits. For $\omega_g < \omega_{g,nom}$ the input constraints are set to

$$\hat{u}_{min} = \begin{bmatrix}\max\left(-\tau_{g0},\, -\tau_{g,rate}\right) \\ \max(-\beta_0,\, -\beta_{rate})\end{bmatrix},$$
$$\hat{u}_{max} = \begin{bmatrix}\min(\tau_{g,max} - \tau_{g0},\, \frac{P_{nom}}{\eta_g \omega_{g0}} - \tau_{g0},\, \tau_{g,rate}) \\ \max(-\beta_0,\, -\beta_{rate})\end{bmatrix},$$

while for $\omega_g \geq \omega_{g,nom}$ the input constraints are set to

$$\hat{u}_{min} = \begin{bmatrix}\min(\tau_{g,max} - \tau_{g0},\, \frac{P_{nom}}{\eta_g \omega_{g0}} - \tau_{g0},\, \tau_{g,rate}) \\ \max(-\beta_0,\, \beta_{rate})\end{bmatrix},$$
$$\hat{u}_{max} = \begin{bmatrix}\min(\tau_{g,max} - \tau_{g0},\, \frac{P_{nom}}{\eta_g \omega_{g0}} - \tau_{g0},\, \tau_{g,rate}) \\ \min(\beta_{max} - \beta_0,\, \beta_{rate})\end{bmatrix}.$$

If the generator speed ω_g is below the rated speed $\omega_{g,nom}$ the pitch angle reference β_c is constrained to its minimum β_{min}. The generator speed ω_g is regulated with the torque reference $\tau_{g,c}$ to the optimal operating point $\omega_{g,ref}$ and vice versa if the generator speed is above the rated speed.

V. SIMULATION RESULTS

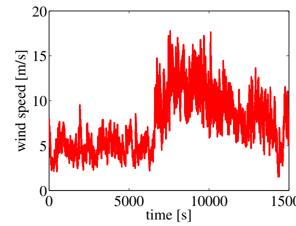
To demonstrate the performance of the wind farm optimisation we present the simulation results of three turbines in a row with FAST [24] and a 80-turbine wind farm adapting the layout of Horns Rev with Aeolus SimWindFarm [21]. We compare the responses of the MPC controller developed in Section IV running at the optimal set points obtained in Section III, the MPC controller with the greedy reference values $\lambda_{r,greedy} = 7.5$ and $\beta_{r,greedy} = 0°$ for every turbine $i \in N$ and the commonly used feedback controller presented in [23]. The simulations are performed for constant wind direction using the model from Subsection II-B and II-C for the wake calculation.

A. Three Turbines in a Row

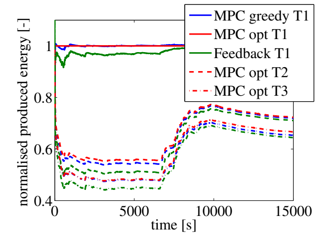
The behaviour of three 5 MW offshore turbines [23] in a row with a spacing of $\bar{s}_{1,2} = \bar{s}_{2,3} = 500$ m is simulated for 15 000 s of real wind data from the Colorado School of Mines and National Renewable Energy Laboratory (see Fig. 4a). The model parameters are the air density of 1.225 $\frac{\mathrm{kg}}{\mathrm{m}^3}$, MPC sampling frequency of 80 Hz, a prediction horizon of 8 steps and the optimal reference values $\lambda_r^* = \begin{bmatrix}7.1 & 7.1 & 7.5\end{bmatrix}$ and $\beta_r^* = \begin{bmatrix}0.8° & 0.8° & 0°\end{bmatrix}$. The ratio of the total power coefficient for greedy and optimal reference values is

$$\frac{C_{P,tot,greedy}}{C^*_{P,tot}} = \frac{1.098}{1.109} = 99.0\%. \tag{13}$$

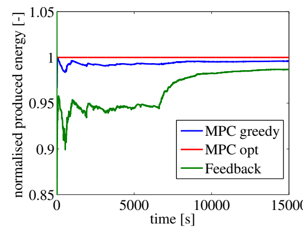
The MPC controller ensures that the wind turbines track the desired reference values. Fig. 4b shows the normalised produced energy of every turbine for the three different controllers. The optimal MPC controller accepts a loss of energy in the first turbine to increase the energy captured by the second and third turbine and achieve a higher energy



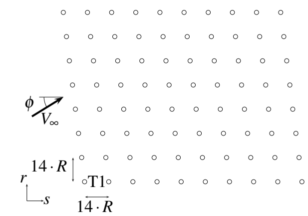
(a) Real wind data used for FAST Simulation.



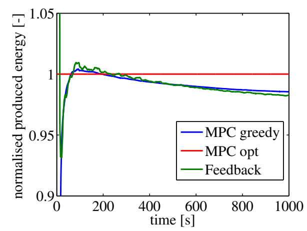
(b) Normalised energy produced by every turbine.



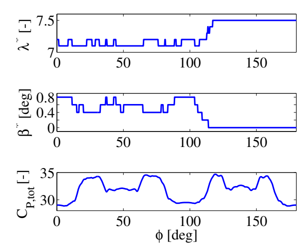
(c) Normalised total energy produced by three turbines in a row.



(d) Layout of the Horns Rev wind farm.



(e) Total energy produced by ten turbines in a row according to Horns Rev layout.



(f) Optimal reference values for turbine T1 and the total power coefficient of the Horns Rev wind farm depending on wind direction ϕ.

Fig. 4: Wind data and results of the simulation performed in FAST with 15000 s of real wind data in Fig. 4a-4c and layout and results of the Aeolus SimWindFarm simulation in Fig. 4d-4f.

output combining all three turbines as visible in Fig. 4c. The feedback controller from [23] captures less power for low wind speeds because it regulates the rotor speed not as consequently to the minimal rotor speed $\omega_{r,min}$. Furthermore, the MPC controllers consider the current wind speed as an additional measurement and react faster to changes in the wind speed. The simulation leads to a ratio of the produced energy of $\frac{E_{tot,greedy}}{E^*_{tot}} = 99.6\%$, where $E_{tot,greedy}$ and E^*_{tot} represent the total energy produced by the greedy, and optimal MPC controllers, respectively, over the simulation time. The gain of energy is not as high as predicted in (13) because both controllers saturate at a power of 5 MW for high wind speed, making optimisation of the power capture impossible.

B. Layout Horns Rev

We adapt the 80 turbine layout from the Horns Rev wind farm in Denmark as in Fig. 4d replacing the Vestas V80-2MW with the 5 MW turbine from [23].

1) Aeolus SimWindFarm: For the simulation with the SimWindFarm toolbox we restricted the wind direction to $\phi = 0°$ (see Fig. 4d). An initial simulation showed that the eight rows do no affect each other. Hence, we reduce the wind farm to ten turbines in a row without loss of generality for computational intensity's sake. The $1000\,\mathrm{s}$ of simulation are performed for a mean wind speed of $10\,\frac{\mathrm{m}}{\mathrm{s}}$, air density of $1.225\,\frac{\mathrm{kg}}{\mathrm{m}^3}$, MPC sampling frequency of $100\,\mathrm{Hz}$ and a prediction horizon of 8 steps. Fig. 4e illustrates normalised total energy production of the ten wind turbines. The optimal MPC controller generates initially less power than the two other controllers, but as soon as the wake reaches the downwind turbines, it starts performing better. The ratio of the power coefficients for greedy and optimal control are $\frac{C_{P,tot,greedy}}{C^*_{P,tot}} = \frac{28.73}{29.03} = 99.0\%$ whereas the simulation leads to a ratio of the produced energy of $\frac{E_{tot,greedy}}{E^*_{tot}} = 98.6\%$. The increase in total energy is achieved by a reduction of the wake effects due to the optimal reference values leading to lower thrust coefficients for the upwind turbines.

2) Optimal reference values for varying wind direction: The optimisation according to Section III can be performed for any wind direction. Hence, we can calculate the optimal reference values for every turbine for a given wind farm layout depending on the wind direction offline and store its values on the wind turbine controller. Fig. 4f shows the reference values and the total power coefficient of the wind farm for turbine T1 (see Fig. 4d) as a function of the wind direction ϕ.

C. Discussion

While we formulated the optimisation problem independent from any wake model, the specific model considered in this work showed an increase in the captured energy of about 1%. However, the possible energy gain depends greatly on the wake model and can go up to 7% as shown in [22].

In reality the wind direction may have fast dynamics but generally with limited amplitude, while the turbine yaw angle dynamics are very slow. The optimisation framework is robust to small changes in the wind direction as the captured power depends on the cosine of the deviation of the wind direction [25] and the optimal reference values are generally not sensitive to wind direction as visible in Fig. 4f. However, to estimate the influence of time varying wind directions further simulations would be necessary accordingly, which cannot currently be performed in Aelos SumWindFarm.

VI. CONCLUSION AND FUTURE WORK

In this paper, we introduced a framework for maximising power output of wind farm by considering wake effects. A static optimisation problem, given a wind farm layout and an arbitrary wake model, was formulated which determined optimal set points for the blade pitch angle and tip speed ratio of individual turbines. In addition, an MPC controller was designed for individual turbines to track the optimal reference values. The approach can be used to provide an upper bound on achievable power of a wind farm regarding the wake effects causing wind deficit. Future directions include the investigation of the influence of time varying wind directions and the extension of the controller to additional objectives such as load mitigation. In addition, to further validate the methodology practical implementations are necessary with measurements on wind tunnels or on real wind farms.

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