Master Project

Tractable Algorithms for Large Scale Mixed Integer Quadratic Programming: A Principal Component Analysis Approach

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Context

In many optimization problems, simple, approximate solutions are more useful than, complex exact solutions. Simplicity is sometimes manifested as sparsity in the solution vector. Sparsity implies that the number of non zero elements is limited. Sparse optimization is useful for: i) eliminating noisy, non informative features, ii) providing intepretability and iii) finding the optimal solution where, nonzero coefficients are limited or have a direct cost (e.g., transaction costs in finance). Because of these features large scale sparse quadratic programming is widely applicable. The standard problem is written in the following form:

$$\begin{aligned} \min_{x} \quad & x^T Q x + c^T x \\ \text{s.t.} \quad & ||x||_0 \leq k \end{aligned} \tag{1}$$

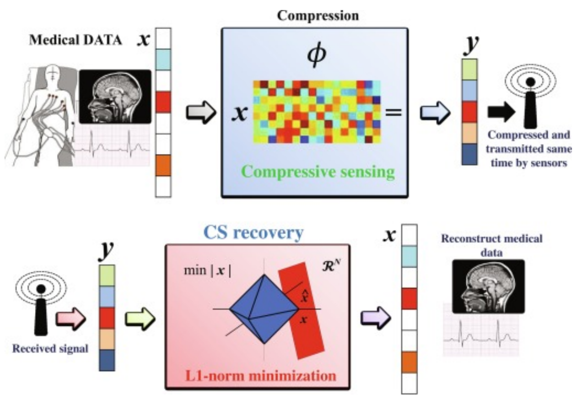

Figure 1: Medical data analysis using sparse regression

Machine learning literature regarding sparse optimization is largely focused around sparse regression. The goal of sparse regression is to retrieve the sparse solution to the problem based on output and input data, an example is displayed in Figure 1. Based on control literature we would also like to highlight MPC as a possible application. We can for example solve a hydro scheduling problem using (1) as a cost function to our MPC problem. Until recently the literature regarding sparse optimization was largely focused on heuristic approaches. Some important advantages of heuristic approaches are feasibility and scalability. However these proxy based methods do have their shortcomings as they do not very well recover the sparsity pattern. For this reason there is an increasing interest in solving the sparse optimization problem directly. The resulting estimators outperform their l_1 counterparts. However these exact methods are limited by their computational time and do not scale very well when the problem size increases.

Project tasks

This master thesis project is aimed at finding a new exact solution to the Sparse Quadratic Programming problem which is not only more largely applicable but also faster than the current best solution for large datasets. To be able to do this the following tasks are proposed:

1. Investigate Sparse Optimization and the different approaches that can be taken to solve this problem
2. Develop an algorithm that can solve the Sparse Quadratic Programming problem
3. Show that the found solution is faster than the state of the art on multiple datasets and sparse components
4. Implement the algorithm in a real world application